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Perceiving Topological Relations

Sami R. Yousif[1] and Elizabeth M. Brannon[2]
[1]Department of Psychology and Neuroscience, University of North Carolina, Chapel Hill, and
[2]Department of Psychology, University of Pennsylvania

Abstract
There are many ways to describe and represent the visuospatial world. A space can be described by its euclidean properties—the size of objects, the angles of boundaries, the distances between them. A space can also be described in nonspatial terms: One could explain the layout of a city by the order of its streets. Somewhere in between, *topological representations*—such as those commonly depicted in public-transit maps—capture coarse relational structure without precise euclidean detail, offering a relatively efficient, low-dimensional way of capturing spatial content. Here, we ask whether human adults quickly and automatically perceive such relations. In six experiments, we show that differences in simple topological features influence a range of visual tasks from object matching to number estimation to visual search. We discuss the possibility that topological relations are a kind of visual primitive that supports visuospatial representation.

Keywords
topology, geometry, perception, spatial cognition, relations

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Introduction

Visual scenes are rich with information. In even mundane scenes, there are dozens of shapes and boundaries and objects that are made up of various contours and edges that can themselves be described with respect to many different geometric properties. How is all of this complex information condensed into a lower-dimensional form? What are the basic building blocks of spatial representation that support scene recognition, object discrimination, and action? (For relevant discussions, see, e.g., Firestone & Scholl, 2014; Vernazzani & Mollo, 2024; Yousif & Keil, 2021; Yousif et al., 2023.)

One classic theory suggests that visual object representations are made up of several discrete components, called *geons* (Biederman, 1987), which are pieced together to form more complex objects. This *recognition-by-components* theory offers a way of understanding how rich, complex images can be represented via a discrete, finite set of parts. All one needs to know is what geons are contained in an image and how they relate to one another (i.e., a lamp is a cone on top of a cylinder), thereby making a seemingly intractable computational challenge relatively straightforward. Other theories posit even more minimal primitive structures. For instance, Julesz (1981) argued for *textons*, in an attempt to explain perceptual sensitivity to features like closure and connectivity. Textons consist of such basic components as "bars" (line segments) and "terminators" (intersections, or junctions; see Julesz, 1982). What both of these theories have in common is an interest in whether visual representations can be plausibly explained by grammarlike representations (e.g., discursive representations that can be combined to create entities of greater complexity, as when combining words into sentences; see Lande, 2024). If visual representations can be described by "image grammars," what might the most basic components of those grammars look like?

Here, we explore the perception and representation of one plausible visuospatial primitive: *topological relations*, or network topology. To understand the basics of network topology, consider the letters "T" and "L"

Corresponding Author:
Sami R. Yousif, University of North Carolina, Chapel Hill, Department of Psychology and Neuroscience
Email: sami.yousif@unc.edu

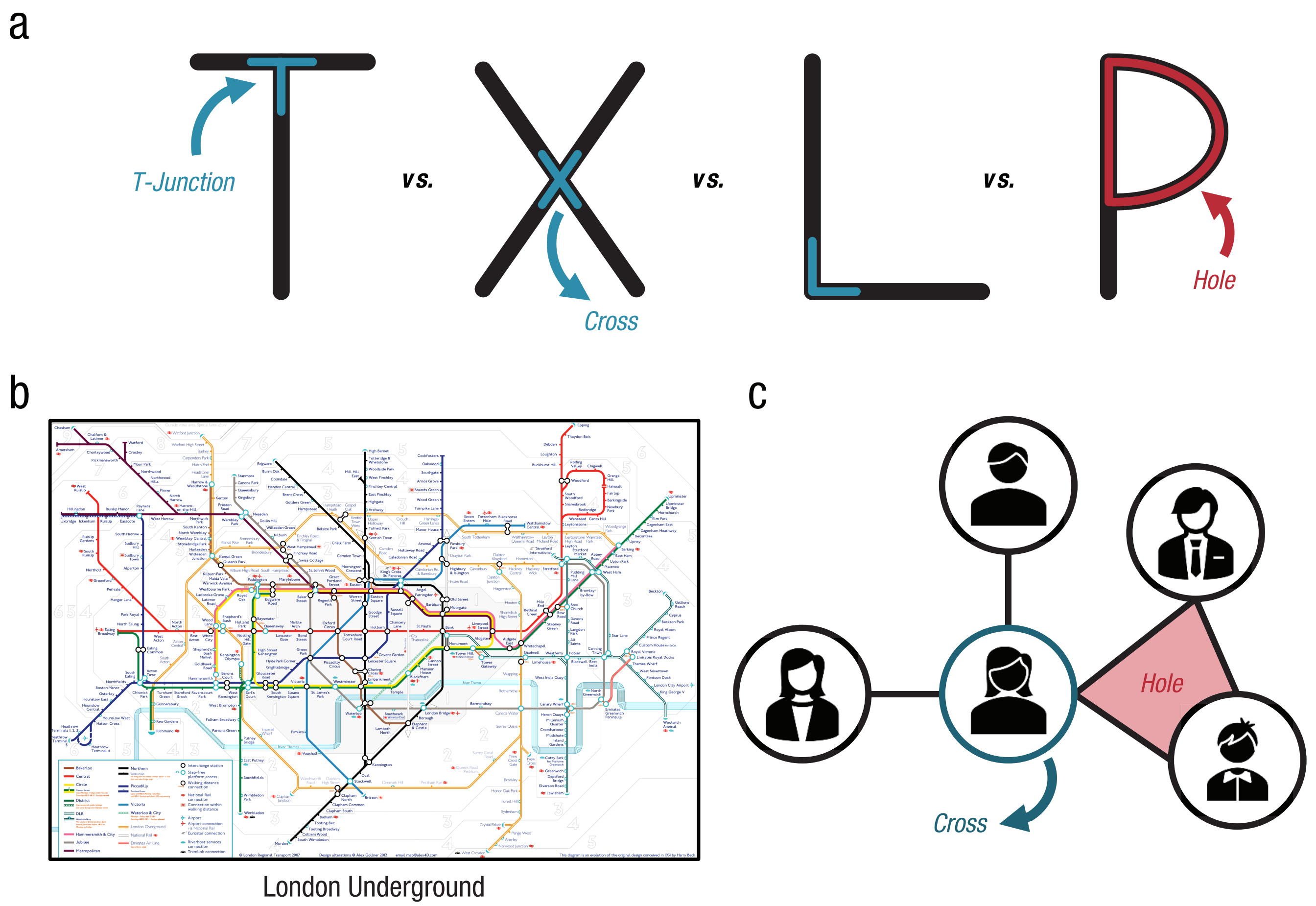

Fig. 1. Topological illustrations. An example of the relative topological parts of the letters "T," "X," "L," and "P" is shown in (a), along with a modern rendition of the original topological map designed for the London Underground (b). In (c), we depict a social network and its constituent topological parts.

(see Fig. 1). Both are (in a certain light) made up of two independent line segments joined at a vertex. But the "L," like a bendable straw, can be bent to form a single line. A "T" cannot be made into a single line, except by way of "cutting"—an indication that it is topologically distinct from an "L." In this way, the "T" has a distinct topological feature (a *T-junction*, or a three-point vertex). Similarly, a "P" can be distinguished from both an "L" and a "T" insofar as it has a hole or a loop in its structure.[1] If this "P" shape were imagined as a maze, maze-goers might find themselves walking around in circles (for more information on these basic features of network topology, see Fig. 1; see also Yousif & Brannon, 2024). This is not just about letters on a page, however; topological relations are functional. Consider the difference between a four-way intersection, a three-way intersection, and a road that merely bends. In the latter case, you are forced to turn; in the former case, you have a choice about where to move. The topological difference in how the roads are joined influences how you would navigate through them.

The investigation of topology, broadly construed, is not new in the field of cognitive science. Prior work has investigated the perception of object topology—for example, how features like holes and closure influence perception of individual objects (Chen, 1982; but see Rubin & Kanwisher, 1985, as well as Chen, 1990). Other research has examined how object topology interacts with other visual and cognitive processes (see Chen, 1985; He et al., 2015; Wei et al., 2019) both in human children (Chien et al., 2012; Dehaene et al., 2006; Kibbe & Leslie, 2016) and in nonhuman animals (Chen et al., 2003). The present work builds on that prior work by examining a different kind of topology—topological relations like T-junctions, crosses, and holes (see Fig. 1; for more information on topological relations and network topology, and how it differs from object topology, see Yousif & Brannon, 2024).

The distinction between object topology and network topology is substantive. Appreciating this more fine-grained distinction may alter how prior work on topological perception is interpreted. Consider, for instance, the work of Wei et al. (2019), who examined how topological changes influence visual working memory. One of the contrasts they drew involved the difference between the letter "E" and the letter "H," which they categorized as a nontopological change because neither letter contains a hole. However, readers can appreciate how, through the lens of network topology, the letters "E" and "H" are quite different: The former has a single T-junction, whereas the latter has two. Is this more subtle difference also one that influences perception and working memory?

In addition, there are theoretical reasons why one might care about the distinction between object topology and network topology: Namely, the "language" of network topological relations offers a low-dimensional format for representing spatial information and therefore may be a crucial basis for cognitive maps (whereas object topology has no bearing on maplike representations; see Figs. 1b and 1c). This is evident in Figure 1b: This map of the London Underground is one of the world's most popular maps, and, when it was first drawn this way about a century ago, it may have been one of the world's first topological maps. Such maps are now common around the world for the simple reason that they are remarkably intuitive. Somehow, such topological renderings of space seem almost more intuitive than depictions that preserve perfect euclidean detail. And this is only the tip of the iceberg: Topological relations may also be sufficient to describe other maplike structures, like social networks (see Fig. 1c). Family lineages, decision trees, networks of interconnected academic papers—all of these things could be described with respect to their topological relations. In this way, examining the representation and perception of basic topological features like holes and T-junctions may shed light not only on the building blocks of visual perception (see also Lande, 2024), but also on the building blocks of mental maps more generally.

The Current Study

We recently showed that human adults are broadly sensitive to several different features of topological networks (Yousif & Brannon, 2024). Here we take the next step to test whether features of topological relations are not just ones that we think about, but ones that are directly perceived. In six experiments we ask whether topological relations (in the form of properties like T-junctions and holes) are processed quickly, automatically, and irresistibly by the visual system.

Research Transparency Statement

General Disclosures

Conflicts of interest: All authors declare no conflicts of interest. **Funding:** This research received no specific funding. **Artificial intelligence:** No artificial-intelligence-assisted technologies were used in this research or in the creation of this article. **Ethics:** This research complies with the Declaration of Helsinki and received approval from a local ethics board.

Study disclosures

Preregistrations, data, and analysis scripts are all available on our Open Science Framework (OSF) page: https://osf.io/3m2ew/

Experiment 1a Disclosures

Preregistration: The research aims and hypotheses and the methods and analysis plan were preregistered (https://aspredicted.org/xnct-mtw5.pdf) on July 4, 2023, prior to data collection, which began that same day. The only slight deviation from the preregistered analyses is that we decided to exclude all responses with a response time greater than 10 s. This choice did not affect the conclusions of the study. All other experiments in this paper had the same exclusion criterion. At a reviewer's request, we also added unregistered analyses comparing the different trial types. **Materials:** The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/ajzb2/files/osfstorage) are publicly available. **Data:** All primary data are publicly available (https://osf.io/4v2gn). **Analysis scripts:** All analysis scripts are publicly available (https://osf.io/tuja8). **Computational reproducibility:** The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 1b Disclosures

Preregistration: The research aims and hypotheses and the methods and analysis plan were preregistered (https://aspredicted.org/ymhq-44n8.pdf) on July 8, 2023, prior to data collection, which began on July 10, 2023. The only slight deviation from the preregistered analyses is that we decided to exclude all responses with a response time greater than 10 s. This choice did not affect the conclusions of the study. All other experiments in this article had the same exclusion criterion. At a reviewer's request, we also added unregistered analyses comparing the different trial types. **Materials:** The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. **Data:** All primary data are publicly available (https://osf.io/j3vq7). **Analysis scripts:** All analysis scripts are publicly available (https://osf.io/tuja8). **Computational reproducibility:** The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 2 Disclosures

Preregistration: The research aims and hypotheses, methods, and analysis plan were preregistered (https://aspredicted.org/pt5g-fvt9.pdf) on August 24, 2023, prior to data collection, which began later that same day. There was one deviation from the preregistered analyses: At a reviewer's request, we also added unregistered analyses comparing the different trial types. **Materials:** The experiment code (https://osf.io/76ngw) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. **Data:** All primary data are publicly available (https://osf.io/yqsju). **Analysis scripts:** All analysis scripts are publicly available (https://osf.io/cg5aj). **Computational reproducibility:** The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 3a Disclosures

Preregistration: The research aims and hypotheses, methods, and analysis plan were preregistered (https://aspredicted.org/gcxs-gvbd.pdf) on October 5, 2023, prior to data collection, which began later that same day. There were no deviations from the preregistered analysis plan. However, we did add a binomial test to support the claim that the majority of participants exhibited the relevant effect. **Materials:** The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. **Data:** All primary data are publicly available (https://osf.io/uxq63). **Analysis scripts:** All analysis scripts are publicly available (https://osf.io/8zkua). **Computational reproducibility:** The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 3b Disclosures

Preregistration: The research aims and hypotheses as well as the methods and analysis plan were preregistered (https://aspredicted.org/gcxs-gvbd.pdf) on October 5, 2023, prior to data collection, which began later that same day. The only deviation from the analysis plan is that we ran a cross-experiment test between Experiments 3a and 3b that was not explicitly preregistered, though we did preregister that we would predict that, if anything, effects in Experiment 3b would be weaker. We also added information about the number of participants who exhibited the relevant effects, with accompanying binomial tests. **Materials:** The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. **Data:** All primary data are publicly available (https://osf.io/u5n89). **Analysis scripts:** All analysis scripts are publicly available (https://osf.io/8zkua). **Computational reproducibility:** The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 4 Disclosures

Preregistration: The research aims and hypotheses, as well as the methods and analysis plan, were preregistered (https://aspredicted.org/qdwp-dnv5.pdf) on December 9, 2023, prior to data collection, which began later that same day. We preregistered that we would run a version of this task with colored pentominoes but subsequently realized that no pattern of results from that experiment would be diagnostic. For that reason, those data were never collected. **Materials:** The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/8wsek/files/osfstorage) are publicly available. **Data:** All primary data are publicly available (https://osf.io/q25z7). **Analysis scripts:** All analysis scripts are publicly available (https://osf.io/u23xc). **Computational reproducibility:** The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiments 1a and 1b: Speeded Comparison Task

First, we investigated whether changes in topology influence participants' ability to rapidly perceive and recognize objects. We created sets of items that each contained a default item and variants of the default item that either maintained or disrupted the object's topology. Participants were given a speeded comparison task in which they were presented with one item and then, after a brief delay, a variant of that item. They were asked to indicate, as quickly as possible, whether the second item was exactly the same as the first one. We predicted that people would be more likely to confuse altered items that maintained topology than altered items that disrupted topology.

Method

For these experiments, and for all subsequent experiments in this article, the sample sizes, primary dependent variables, and key statistical tests were chosen in advance and were preregistered (see https://osf.io/3m2ew).

Participants. One hundred participants were tested online via Prolific—50 in Experiment 1a and 50 in Experiment 1b. The exclusion criteria we preregistered were highly conservative, so no participants were excluded. This study was approved by the University of Pennsylvania Institutional Review Board.

Stimuli. For each experiment, we created 20 distinct item sets. For Experiment 1a, each item set consisted of four stimuli: (a) a default item, which consisted of a few line segments combined to form a letterlike object; (b) a modification of the default item in which a single line was rotated or shifted by a fixed amount (but without altering the original topology); (c) a modification of the default item in which the same line as in (b) was rotated or shifted in the opposite direction so that the resulting object had a distinct topology); and (d) a modification of the default item in which the same line as in (b) was rotated or shifted by twice as much as for the previous items (but, again, without altering the original topology). An example of an item set with each of its four stimulus variants can be seen in Figure 2a. Some item sets involved rotation, and others involved shifting of a line segment,

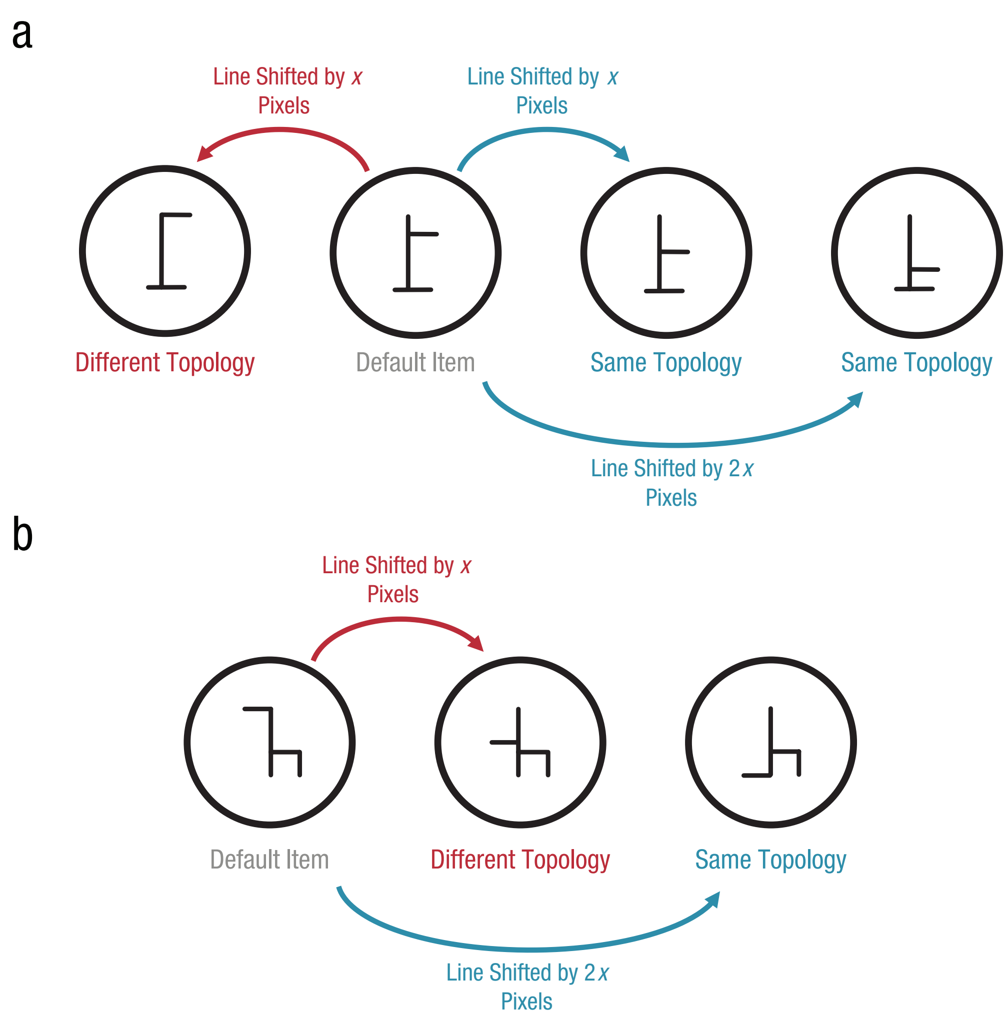

Fig. 2. A visual explanation of the stimulus design for the stimuli used in Experiments 1, 2, and 3. Each stimulus set used in Experiment 1a consisted of four possible items (a). To construct each stimulus set, we started with a default item and then modified it by translating or rotating one line. In this example, one line was translated upward in a way that altered the topology (left), one was translated downward in a way that preserved the topology (inner right), and one was translated downward by twice as much in a way that still preserved the topology (right). Each stimulus set used in Experiments 1b, 2, and 3 consisted of three different items (b). To construct each set, we started with a default item and then modified it by translating or rotating one line. In this example, the leftmost line was translated downward in a way that altered the topology (middle) and translated downward by twice as much in a way that preserved the original topology. In this way, for all stimulus sets, we effectively had items that were topologically matched but twice as different with respect to their objective euclidean properties.

but no set involved both. Some item sets involved topological changes in T-junctions, others involved changes in holes, and some involved both.

We created 20 new item sets for Experiment 1b with the same general design but with one significant change. Each set contained only three variants: a default item, one with a matching topology, and one with a distinct topology. The key difference between this version and the previous version is that the changes to the topology-matched item occurred in the same direction as the changes to the topology-mismatched item. In other words, if a line was shifted 20 pixels to the left to create the topology mismatch, it would be shifted 40 pixels to the left to create the topology match. In this way, the topology mismatch was always objectively more similar to the default item compared to the topology-matched item. An example of an item set can be seen in Figure 2b. For both Experiments 1a and 1b, the 20 distinct item sets were created to have as much variety as possible. The full stimulus sets can be accessed on our OSF page.

Procedure. Participants were told that they would see letters from a fictional language and that they needed to compare the letters to identify whether they were "exactly the same" or not. On a given trial, one of the 20 unique default items was presented for 1 s followed by a 1-s delay. Subsequently, one of the other stimuli from that set was presented, and participants were asked to indicate as quickly as possible whether the second item was the same (by pressing "s") or different (by pressing "d") from the item they had first seen. In Experiment 1a, the second item was identical to the first 40% of the time (i.e., it was the default item). The three possible variations of the default item (see Fig. 2a) were each presented on 20% of the trials. Thus, the correct response was "same" on 40% of trials and "different" on 60% of trials. Each unique combination of item set and trial type was presented twice, resulting in a total of 200 trials (20 unique item sets × [4 trial types + 1 extra iteration of the same trials] × 2 repetitions). In Experiment 1b, there were only three trial types (corresponding to the three item types; see Fig. 2b), and they were presented with equal frequency. Thus, the correct response was "same" on 33% of trials and "different" on 66%. Each unique combination of item set and trial type was presented twice, resulting in a total of 120 trials (20 unique item sets × 3 trial types × 2 repetitions).

In both experiments, the order of the trials was fully randomized for each participant. In these and all subsequent experiments, participants completed two representative practice trials prior to beginning the task.

The items were presented in a display of circles of two rows and three columns (see Fig. 3a). The first item was presented randomly in one of the six locations. The second item was presented randomly in one of the other five locations.

Results

Results can be seen in Figures 3b (Experiment 1a) and 3c (Experiment 1b). In accordance with the preregistered analysis plan, we tested the preregistered predictions that participants would be (a) less accurate and (b) slower to identify a different item as different when it shared topology with the default item. Said another way, we predicted that participants would mistake topology-matched (but physically different) items as the same more often than topology-mismatched items. For Experiment 1a, we first compared the two trial types for which there was an equal amount of physical change (but a difference in topology).[2] As is evident from Figure 3b, participants were significantly more likely to falsely indicate that an item was the same as the original item when it retained the topology of that item ($M_{p\text{-diff}} = 0.68$, $SD = 0.15$) compared with when it did not ($M_{p\text{-diff}} = 0.94$, $SD = 0.06$), $t(49) = 13.34$, $p < .001$, $d = 1.89$. Participants were also slower to correctly classify items as different when those items maintained topology of the original item—response time (RT): $M = 1{,}406$ ms, $SD = 397$ ms—as opposed to ones that did not ($M = 1{,}295$ ms, $SD = 335$ ms), $t(49) = 4.73$, $p < .001$, $d = 0.67$. Strikingly, Figure 3b also shows that participants were more likely to falsely indicate that an item that retained the original topology was the same, even when it changed physically by twice as much ($M_{p\text{-diff}} = 0.91$, $SD = 0.08$) as items with altered topology, $t(49) = 3.20$, $p = .002$, $d = 0.45$. However, we found no difference in response time ($M = 1{,}268$ ms, $SD = 319$ ms), $t(49) = 1.27$, $p = .21$, $d = 0.18$.

In response to a reviewer's question, we ran an additional unregistered analysis on accuracy to ask whether the findings differed for holes and T-junctions. The difference between the topology mismatch and the equivalent topology match was significant (and sizeable) whether we analyzed only those trials in which holes varied, $t(49) = 9.10$, $p < .001$, $d = 1.29$, in which T-junctions varied, $t(49) = 8.84$, $p < .001$, $d = 1.25$, or both, $t(49) = 12.65$, $p < .001$, $d = 1.79$. In fact, this effect was in the correct direction for all 20 items tested; 17 of these 20 tests were independently significant.

For Experiment 1b, participants were again significantly more likely to falsely indicate that an item was the same as the original item when it maintained the topology of that item ($M_{p\text{-diff}} = .88$, $SD = .11$) compared with when it did not ($M_{p\text{-diff}} = 0.94$, $SD = 0.06$), $t(49) = 4.21$, $p < .001$, $d = 0.60$. They were also slower to correctly identify items as different when the items retained the topology of the original item (RTs: $M = 1{,}356$ ms,

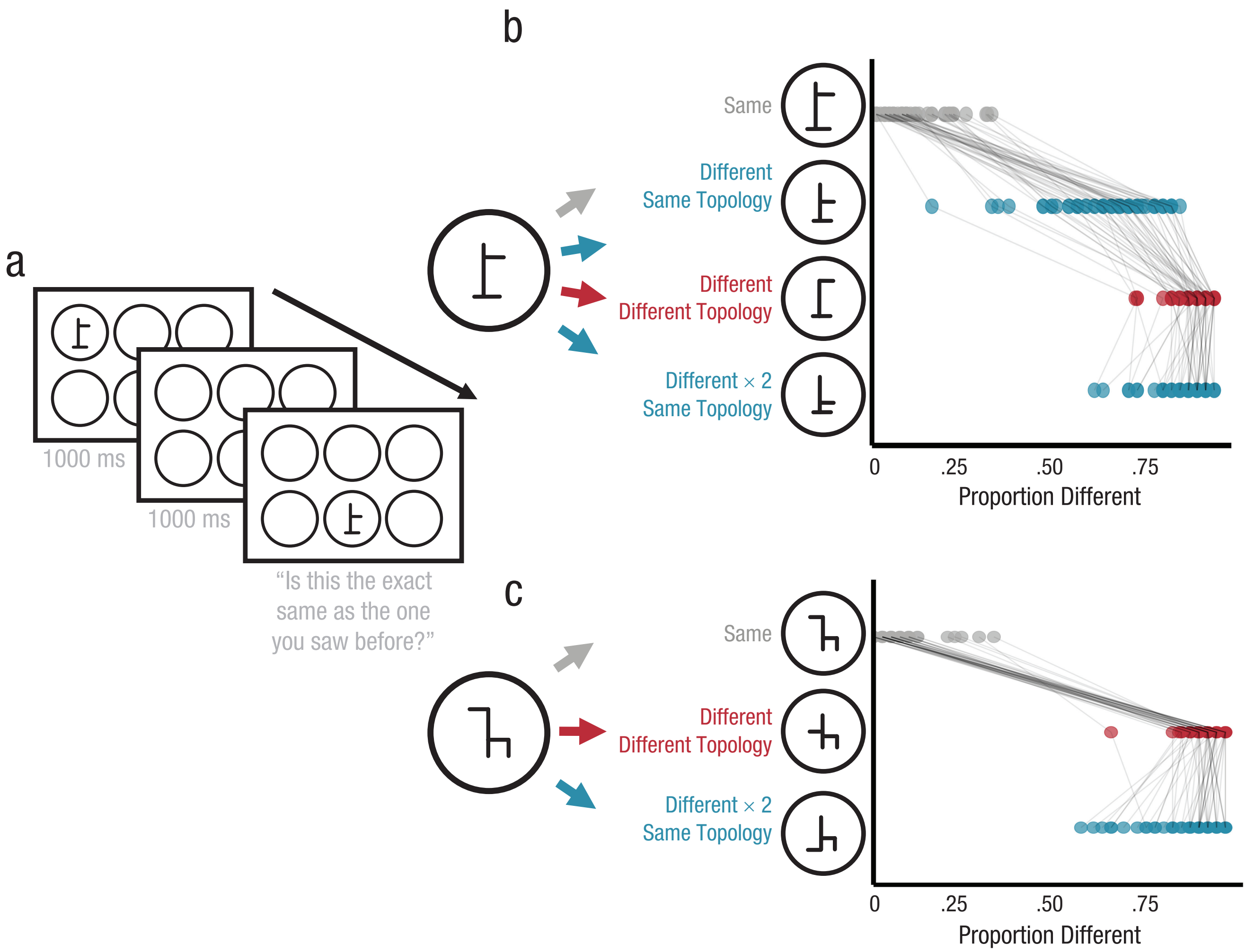

Fig. 3. A simple depiction of the speeded comparison task (a). The results of Experiment 1a are shown in (b) and the results of Experiment 1b in (c).

SD = 394 ms) compared with when topology was altered (*M* = 1,296 ms, *SD* = 328 ms), $t(49) = 2.54$, $p = .014$, $d = 0.36$. In response to a reviewer's question, we conducted another unregistered analysis on accuracy to assess whether the effect held for both topological relations. The effect was significant for the sets that involved changes to holes, $t(49) = 4.95$, $p < .001$, $d = 0.70$, and T-junctions, $t(49) = 2.35$, $p = .023$, $d = 0.33$), but marginal for those trials that involved a change to both, $t(49) = 2.01$, $p = .05$, $d = 0.28$. We attribute this to the fact that the sets that involved a change to both were generally more complex. Combined, the results of Experiment 1a and 1b suggest that simple topological changes have a dramatic influence on people's ability to quickly perceive and identify objects, despite the fact that the topological matches were, from a euclidean perspective, sometimes objectively less similar to the original item than the nonmatching alternative.

Experiment 2: Visual Search

Next, we investigated whether these same sorts of topological features influence the ability to attentionally select for objects in a display. An abundance of prior research demonstrates that when adults are tasked with locating a target stimulus embedded in an array of distractors, similarity between the distractors and the target slows target selection (Treisman & Gormican, 1988; Treisman & Souther, 1985; for reviews, see Eckstein, 2011; Wolfe & Horowitz, 2017). Accordingly, we asked whether similarity in topological features would likewise influence visual search.

Method

Fifty participants completed the task online via Prolific. A single participant was excluded and replaced because of overt negligence or inattention (but those data were still included, with a note, on our OSF page). Individual response times greater than 10 s were excluded. The task used the same stimuli as Experiment 1b.

On each trial, participants were shown a target item for 2.5 s followed by an array of 16 items presented in random orientations. The target was present in the test array on half of the trials. All items other than the target were identical to each other (but in a random orientation). Participants were instructed to press "Y" if the target was present in the display and "N" if the target was not present.

As described for Experiment 1b, each item set consisted of three items: a default item, an item with altered topology, and a third item with the same topology as the default item but with a physical alteration that was twice as large as the alteration to the topology mismatch. The target item on each trial was always either the default item (see Fig. 2b, left) or the topology match (see Fig. 2b, right). The topology mismatch (see Fig. 2b, center) was never the target item. The distractor items, therefore, could either be the topology mismatch, or whichever of the other two items was not the target. Thus, for half the target-present trials the distractors were topologically distinct and on the other half topologically identical to the target. The topology mismatch was always objectively more like the target than the topology match (see Fig. 4a for an example).

Each participant was tested on 160 trials, counterbalanced in the following way: 20 item sets × 2 variations as the initial target (i.e., the default item vs. the topology match) × 2 trial types (distractors of the same topology or of a different topology) × 2 (whether the target is present or absent in the display). The order of the 160 trials was fully randomized for each participant.

Results

The results of Experiment 2 are summarized in Figure 4. We predicted that search times would be longer when the distractor objects had the same topology as the target compared with when the distractor items had a different topology from the target. This pattern held for both target-present trials, $t(49) = 5.08$, $p < .001$, $d = 0.72$. and target-absent trials, $t(49) = 10.14$, $p < .001$, $d = 1.43$. For target-present trials, topology did not influence accuracy, $t(49) = 0.98$, $p = .33$, $d = 0.14$. However, when the target was absent, participants were significantly more likely to press "yes" when the distractors were of the same topology ($M_{p\text{-yes}} = 0.26$, $SD = 0.15$) than of a different topology ($M_{p\text{-yes}} = 0.08$, $SD = 0.10$), $t(49) = 11.62$, $p < .001$, $d = 1.64$. These results are depicted in Figure S1 in the Supplemental Material available at https://osf.io/3m2ew/.

We once again conducted unregistered analyses to assess whether the response time difference held across all item types (those which involved a change in holes, T-junctions, or both). The effect was significant for the sets that involved changes to holes, $t(49) = 5.98$, $p < .001$, $d = 0.85$, and T-junctions, $t(49) = 2.41$, $p = .020$, $d = 0.34$, but it was not significant for trials that involved a change to both, $t(49) = 1.63$, $p = .11$, $d = 0.23$.

Topology meaningfully influenced participants' ability to attentionally select items in a display. Participants found it more difficult to identify a target in a sea of

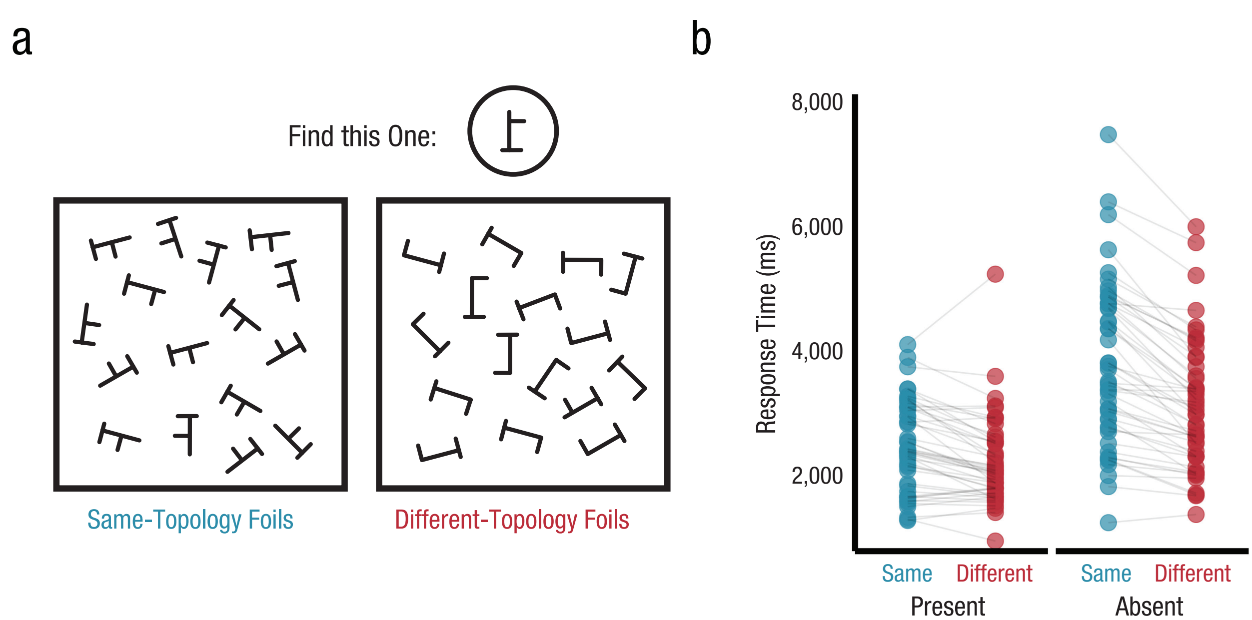

Fig. 4. A simple depiction of the visual-search task (a) and the results of Experiment 2 (b).

distractors that had the same topology as the target (compared with an altered topology) even though the topologically mismatched distractors were always objectively more similar to the targets than the same-topology distractors. These results suggest that topological relations, over and above pixel-level similarity, influence attentional selection.

Though not examined here, prior work has (perhaps unsurprisingly) shown that search is also sensitive to topological features such as holes (Elder & Zucker, 1994; Treisman & Gormican, 1988), a finding that is consistent with the present view. We also note that these findings are potentially inconsistent with prior work that has demonstrated limited sensitivity to crosses versus T-junctions, a difference that is topologically relevant (see Cheal & Lyon, 1992; Wolfe & DiMase, 2003). However, this prior work may suggest that it is the difference between two-point vertices (L-junctions) and three-point vertices (T-junctions) that is most readily represented by the visual system and that n-point vertices greater than three are less well represented in comparison. To fully understand the primitives of visual perception, future researchers should explore this question.

Experiments 3a and 3b: Number Comparison

Topological features like T-junctions and holes are meaningful because they are functional. The letter "T" and the letter "L" are similar in that they can be thought of as two adjacent lines. Yet the "L," unlike the "T," is (topologically) equivalent to a straight line. You can think of the lower stem of the "L" as a bend in a straw, for instance. In this way, the "L" shape, but not the "T" shape, could be perceived as a single line. Therefore, we wondered whether simple topological differences (in this case, differences in the numbers of T-junctions) might influence number estimation. We asked, for instance, whether a display of "L" shapes will be perceived as having fewer overall line segments than a carefully matched display of "T" shapes (because "L" shapes are topologically equivalent to a straight line, whereas "T" shapes cannot be reduced to a single line). We tested two different number-comparison tasks: One in which participants were asked to compare the total number of line segments across images, and one in which participants were asked to compare the total number of objects.

Method

Two groups of 50 participants (Experiment 3a and Experiment 3b, respectively) completed the tasks online via Prolific. We used six of the 20 stimulus sets from Experiment 1b (those that featured only a change in a single T-junction). In Experiment 3a, participants were presented with two arrays of items side by side for 1 s and asked to indicate which array had more total line segments. To help participants understand the task, they were first shown an image with six distinct line segments. Then they were shown an image with the same six line segments but overlapping in pairs to form three Xs. It was then explained that this would still count as six distinct line segments. Finally, the same image with three Xs was shown with each of the six line segments in a distinct color, to ensure that participants understood what each individual line segment looked like. Participants were instructed to make their judgments throughout the task accordingly. Each array was composed of a single item presented in random locations and orientations (see examples in Fig. 5a). Participants were asked to press "Q" if the left side had more line segments and "P" if the right side had more line segments.

On each trial, participants compared an array composed of items from one of the two topology-matched cases (i.e., "L" or reverse "L") and another set composed of the nonmatched case (i.e., upside-down "T").

There were a total of 168 trials, counterbalanced in the following way: There were 6 item sets × 2 variations as the initial target (e.g., "L" and reverse "L") × 2 sides (left or right) × 7 possible number comparisons ([10, 16], [22, 16], [16, 16], [16, 16], [16, 16], [16, 22], [16, 10]). The order of the 168 trials was fully randomized for each participant.

Experiment 3b was exactly the same as Experiment 3a except that participants were asked to compare the total number of objects rather than the number of line segments. They were given the same instructions except that they were explicitly told that the three Xs should be counted as three total objects rather than six total line segments.

Results

The results of Experiments 3a and 3b can be seen in Figure 5. Our analyses focused on trials in which both arrays contained 16 objects (and thus an equal number of line segments and objects). In Experiment 3a, when participants were asked to compare the number of overall line segments, participants selected the side with more T-junctions as having more segments the majority of the time, $M_{p\text{T-junc}} = 0.63$, $SD = 0.08$, $t(49) = 12.35$, $p < .001$, $d = 1.75$. This was independently true for five of the six unique item sets tested (ps < .001), but not for the sixth ($M_{p\text{T-junc}} = 0.51$, $SD = 0.13$), $t(49) = 0.62$, $p = .54$, $d = 0.09$. Of the 50 participants tested, 49 of 50 selected the side with more T-junctions more often than not (binomial test, $p < .001$).

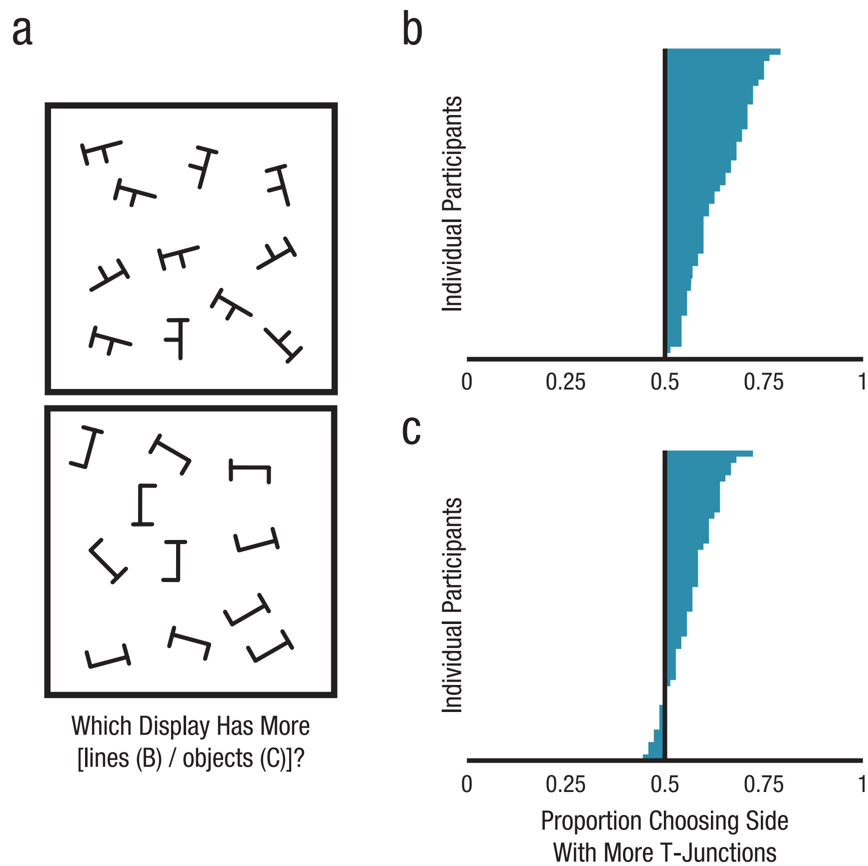

Fig. 5. Experiments 3a and 3b. In (a) we show a simple depiction of the number-comparison task. The results of Experiment 3a, broken down by participant, are illustrated in (b), and the results of Experiment 3b, broken down by participant, are illustrated in (c).

In Experiment 3b, when participants were asked to compare the total number of objects, participants also selected the side with more T-junctions as more numerous the majority of the time, $M_{p\text{T-junc}} = 0.56$, $SD = 0.07$, $t(49) = 6.96$, $p < .001$, $d = 0.98$. This was independently true for only three of the six unique item sets tested (*p*s < .05), and one of these tests would not survive Bonferroni correction. Still, of the 50 participants tested, 38 of 50 selected the side with more T-junctions (binomial test, $p < .001$).

Though this step was not preregistered, we ran a two-sample *t* test comparing overall performance on the two tasks. As expected, participants were significantly more likely to choose the side with more T-junctions when asked to evaluate the number of line segments compared with the number of objects, $t(98) = 4.83$, $p < .001$, $d = 0.97$. These results are consistent with prior work demonstrating that topological features like connectedness influence number estimation (Franconeri et al., 2009; He et al., 2015). Here, however, we have shown that number estimation is influenced by more subtle topological features like the presence or absence of a T-junction. These features are more subtle in the sense that they inherently involve connection; both a "T" and an "L" contain two lines joined at a vertex. Yet it seems to matter not only whether two objects are connected, but how. Although the effect was much more pronounced when people compared the number of line segments (as expected), the fact that there is still a slight bias in favor of the items with more T-junctions even when participants compare the total number of objects may suggest that the visual system cannot help but automatically, irresistibly perceive the number of individual line segments (just as the visual system cannot help but experience a wide variety of other spatial and numerical illusions, including other illusions caused by topological differences; see Franconeri et al., 2009; He et al., 2015). These findings also demonstrate that topological relations influence not only the ability to rapidly perceive and identify objects (as in the prior experiments), but also the ability to perceive other information about them, such as their number.

Experiment 4: Shape-Combination Task

The visual system tracks not only what is perceived in the present, but what is possible in the future. When navigating a busy highway at high speeds, for instance, it may benefit a driver to anticipate the visual world several seconds in advance—which cars might speed up, slow down, or swap lanes. Guan and Firestone (2020) demonstrated that the visual system also makes predictions about simple shapes and how they may combine. Inspired by this approach, we had participants complete a shape-combination task in which they initially see two distinct pentominoes (shapes resembling Tetris pieces but made up of five blocks instead of four) and then one image consisting of 10 blocks. Participants were asked on each trial to indicate whether the 10-block test image was a combination of the original two pentominoes or not. Critically, the pentominoes were combined in such a way that the 10-block test stimulus preserved or altered the topology of the original pentominoes. We asked whether participants would find it more difficult to identify a combined object with altered topology than a combined object which neither added nor removed any topological features.

Method

Fifty participants completed the task online via Prolific.

Stimuli. We created 11 unique pentomino stimuli, each composed of five equisized blocks. (Pentominoes resemble tetrominoes from the popular game Tetris, but whereas tetrominoes are made up of four blocks each, pentominoes are made up of five blocks each.) From these 11 unique pentomino stimuli, we created 10 unique pairs of two pentominoes. Individual pentominoes could be reused, but no pair was repeated. For each unique pair of pentominoes, we created three types of test stimuli. Two of the types were combinations of the two pentominoes, combined in such a way that at least one of the five blocks from each shape was adjoined (see Fig. 6a). For one of these types (same pentominoes–same topology), the pentominoes were combined in such a way that no topological feature was added or removed; for example, if the two shapes individually had three total T-junctions, then the combined shape would also have three total T-junctions. (The combination of the pentominoes could result in the addition of a L-junction, but through the strict lens of network topology, L-junctions are not topologically relevant. Our stimuli also test the bold prediction that people are not only sensitive to T-junctions, but that they are more sensitive to T-junctions than to L-junctions.) For another of these types (same pentominoes–different topology), the pentominoes were combined in such a way that at least one topological feature was added or removed; for example, if the two shapes individually had zero holes, then the combined shape might have one. For the remaining type (different pentominoes–different topology), the object was a random collection of ten blocks—not a combination of any two pentominoes. (This latter type existed so that there were trials on which participants should respond "no"; see the Procedure section.) There were two exemplars of each of the three trial types for each unique pair of pentomino stimuli. For the first two combination types (i.e., those that were a true combination of the previously seen pentominoes), there were two distinct test images. For one of the two, the featured pentominoes were presented in the same orientation in the test display as in the original initial display; for the other, at least one of the pentominoes were rotated. A visual example of all 11 unique pentominoes and one combination, with all six trial types, can be seen in Figure S2 in the Supplemental Material. All of the stimuli were rendered in 3D as slate-gray blocks on a gray background, complete with depth and shadow.

Procedure. Participants were presented with two different pentominoes side by side for 2 s. After a 1-s delay a single central test image was displayed, and participants were instructed to press "S" if they judged the test stimulus to be a combination of the two original pentominoes; they were to press "D" if they judged the test stimulus to be distinct from the original two pentominoes. For two thirds of the trials, the test stimulus was indeed a true combination of the original pentominoes (see Fig. 6a).

There were a total of 120 trials, counterbalanced in the following way: 10 unique pentomino combinations × 3 trial types (see the Stimuli section) × 2 sides (each pentomino of each pair was shown once on the left and once on the right) × 2 combination types (with rotation or without rotation). For the trials in which the test item was not a combination of the previously seen pentominoes, we created two different versions of the trials just to keep the trial types parallel. The order of the 120 trials was fully randomized for each participant.

Results

A summary of the results can be seen in Figure 6b. Per our preregistered analysis plan, our key question was whether participants would be slower, less accurate, or both in identifying combined pentominoes as the same when the combination disrupted topology. As predicted, participants were more likely to falsely indicate that the combined items were different when the topology changed ($M_{p\text{-diff}}$ = 0.25, SD = 0.12) compared with

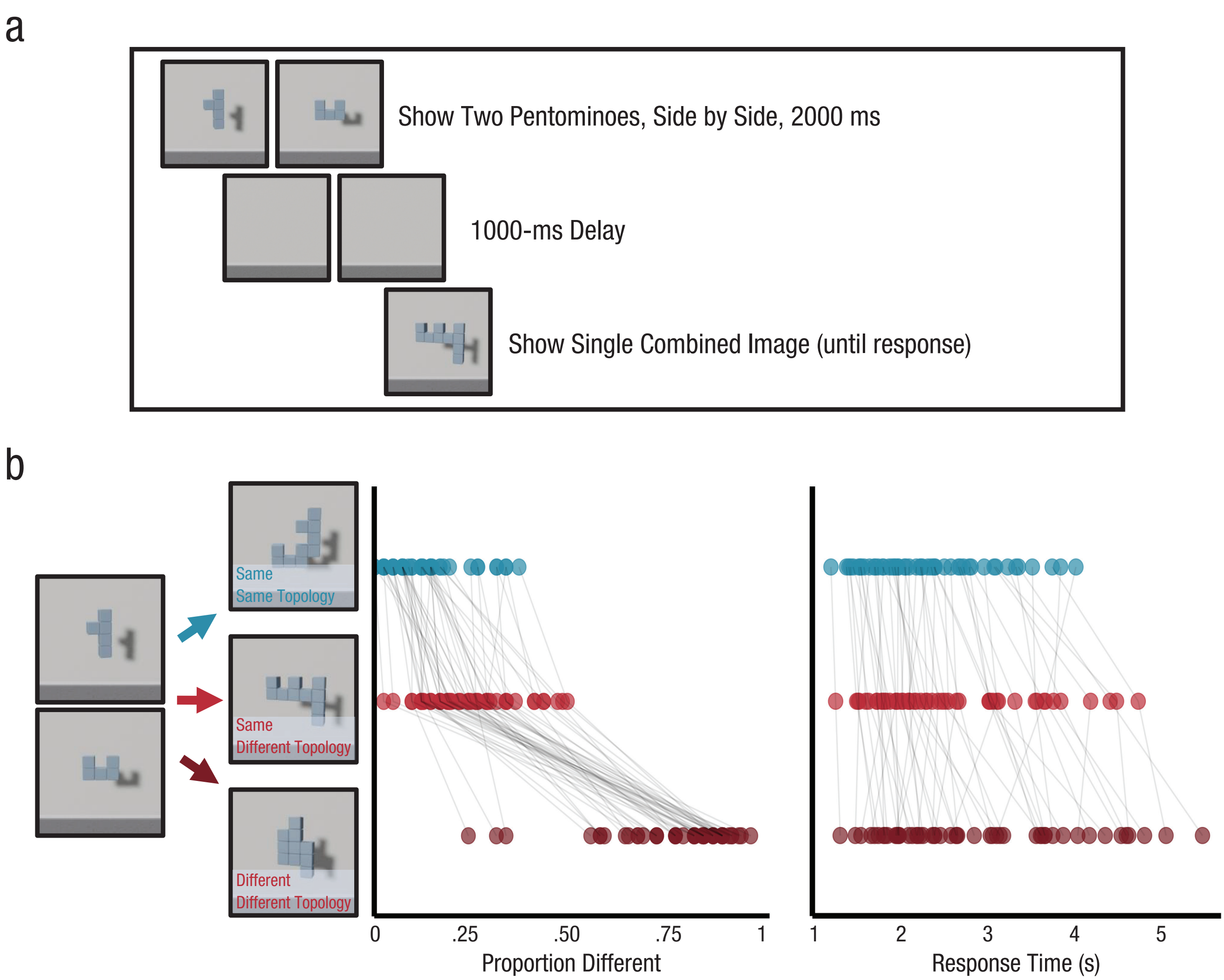

Fig. 6. Experiment 4. In (a) is shown a simple depiction of the shape-combination task; in (b) are shown the overall results of Experiment 4, including response (same or different) as well as response time.

when it was maintained ($M_{p\text{-diff}}$ = 0.13, *SD* = 0.10), $t(49)$ = 9.28, $p < .001$, d = 1.31. Additionally, participants were slower to respond when the topology changed (*M* = 2,545 ms, *SD* = 877 ms) than when topology was maintained (*M* = 2,281, *SD* = 702 ms), $t(49)$ = 5.25, $p < .001$, d = 0.74.

We next conducted an item-level analysis. There were 10 unique pentomino combinations. Of those ten, nine of them exhibited an accuracy effect in the same direction as the overall result; six of these were independently significant (without any correction; *p*s < .05). Similarly, the same nine exhibited an RT effect in the same direction as the overall result; five of these were independently significant (without any correction; *p*s < .01). Note that these results do not reflect a speed accuracy trade-off. People are simultaneously less accurate and slower on combined "same" trials in which the topology changes. This was independently true for nine of the ten items tested, and statistically true for many of them. This is especially impressive considering that each unique item-type and trial-type combination was seen only four times, leaving considerable room for noise.

General Discussion

Evidence from six experiments and four unique paradigms suggests that features of network topology pervade perceptual processing. Subtle changes in a structure's topological relations systematically influenced participants' ability to rapidly identify those forms (Experiment 1 and Experiment 4), rapidly localize them in a crowded display (Experiment 2), and even enumerate their constituent parts (Experiment 3). These results suggest that people are not only sensitive to topological relations in deliberate, cognitive tasks (see Yousif & Brannon, 2024) but that they may perceive topological relations directly.

These findings reflect more than a peculiar quirk of the visual system. The topological forms studied here are functional. Consider, for instance, the finding that objects with more T-junctions are perceived as having more individual line segments than equivalent objects with fewer T-junctions (Experiment 3). We made that prediction because a T-junction, compared with an L-junction, represents a functional part of a space—a turn in a maze, a decision to be made, a vertex than cannot be bent away. A snake might temporarily find itself in the shape of an "L," but it could never find itself in the shape of a "T." If you see a snake in the shape of a "T," that is either a two-headed snake, or two snakes. (Either way, best to stay away!) Perhaps it is important for our visual system to represent the possibility that an "L" shape can readily be transformed into a straight line—that an "L" is more consistent with "oneness" than "two-ness" (and that a "T" shape, in contrast, is more consistent with two-ness). Holes are functional in a similar way: If you find yourself navigating in a maze with many hole structures, you are likely to find yourself going in circles. Therefore, sensitivity to these seemingly arbitrary forms may reveal something about what kinds of structure the visual system cares to encode in the first place.

Relation to other research

Prior work has demonstrated perceptual sensitivity to features of topology, such as closure and in/out relations (see Chen, 1982; Lovett & Franconeri, 2017). However, the sort of topology studied here—network topology, or topological relations—is distinct from what has been studied in earlier research. As discussed in the introduction, prior work on topological representation has, in its focus on object topology, glossed over the sorts of topological features studied here (see, e.g., Wei et al., 2019) and thus may be representing an incomplete view of the extent to which topology influences perception. Yet perhaps the main reason to care about the difference between object topology and the sort studied here is that topological relations could plausibly form the basis of cognitive maps (see Figs. 1b and 1c), and, in this way, should be studied as a potentially crucial component in cognitive maps. Insofar as T-junctions and holes and crosses can be combined like building blocks, they are useful primitives: From these simple parts may arise representations of infinite complexity. Perhaps for this reason some work points to the possibility of topological spatial representations (see, e.g., Coutrot et al., 2022; Epstein et al., 2017). In fact, although not described in exactly this way, there is evidence consistent with this possibility: One classic finding in spatial cognition is that, after navigating an unfamiliar environment, people consistently misrepresent all turns in that environment as if they were 90° angles (even those that differed substantially from 90°; see Byrne, 1979; Moar & Bower, 1983). This suggests that people represent the existence of a turn, but not its magnitude. This makes sense through the lens of network topology, as precise angles and distances are not topologically relevant.

The present work also offers a way to search for additional primitives underlying human spatial representations (see Yousif, 2022). Prior work has suggested that human spatial representations are fundamentally euclidean in nature, relying on representations of properties like length, distance, and angle (see Dehaene et al., 2006; Izard & Spelke, 2009 Izard et al., 2011; Lee et al., 2012; Yousif & Lourenco, 2017). More recent work has speculated about geometric primitives that may support spatial representation, like spirals (Sablé-Meyer et al., 2022). Here, we have shown that topological relations (though not mutually exclusive with the previous alternatives) are another possibility. Specifically, we have shown that people rapidly and automatically perceive topological relations, independently from other euclidean features. The relation between geometric primitives and topological primitives is worthy of further investigation.

Additionally, the results presented here may shed light on the representational primitives that support visual perception. Our work offers a framework through which to interpret prior findings emphasizing the importance of "terminators" (Julesz, 1982), line endings (Cheal & Lyon, 1992), and intersections (Wolfe & DiMase, 2003) in visual perception. Perhaps the unifying factor in all of these cases is a sensitivity to topological relations. Topological features like T-junctions and holes may also bear on the study of *shape skeletons*, which have been studied extensively for their possible role in shape perception and object representation (see, e.g., Ayzenberg & Lourenco, 2019; Ayzenberg et al., 2019, 2022; Firestone & Scholl, 2014). After all, the structure of shape skeletons can be naturally described in topological terms, and that topological structure may (or may not!) be a critical part of how that skeleton is constructed or represented (see Green, 2023). Future work should investigate whether, or how, topological-skeletal representations may support not only object recognition, but also the representation of relational information in many distinct domains (e.g., social networks, decision trees).

Limits on generalizability

These experiments were conducted on educated, Western, English-speaking adults, and consequently we do not make any claims about generalizability beyond that demographic. It remains an open question to what extent the sort of topological sensitivity studied here applies to all populations.

Conclusion

Our findings reveal that people are not only broadly sensitive to topological relations, but that perception itself relies on these representational forms. In fact, we have described a few cases in which the perception of network topology is actually more salient than euclidean geometric properties. This remarkable sensitivity to topological relations suggests that they may be among the building blocks of visual and spatial representation.

Transparency

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Author Contributions
Sami R. Yousif: Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Project administration; Resources; Software; Validation; Visualization; Writing – original draft; Writing – review & editing.
Elizabeth M. Brannon: Conceptualization; Funding acquisition; Writing – review & editing.
Declaration of Conflicting Interests
The author(s) declared that there were no conflicts of interest with respect to the authorship or the publication of this article.
Artificial Intelligence
No artificial-intelligence-assisted technologies were used in this research or in the creation of this article.
Ethics
This research complies with the Declaration of Helsinki (2023) and received approval from a local ethics board.
Open Practices
For all studies, preregistrations, data, and analysis scripts are all available on our Open Science Framework (OSF) page: https://osf.io/3m2ew/

Experiment 1a disclosures. Preregistration: The research aims and hypotheses and the methods and analysis plan were preregistered (https://aspredicted.org/xnct-mtw5.pdf) on July 4, 2023, prior to data collection, which began that same day. The only slight deviation from the preregistered analyses is that we decided to exclude all responses with a response time greater than 10 s. This choice did not affect the conclusions of the study. All other experiments in this paper had the same exclusion criterion. At a reviewer's request, we also added unregistered analyses comparing the different trial types. Materials: The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/ajzb2/files/osfstorage) are publicly available. Data: All primary data are publicly available (https://osf.io/4v2gn). Analysis scripts: All analysis scripts are publicly available (https://osf.io/tuja8). Computational reproducibility: The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 1b disclosures. Preregistration: The research aims and hypotheses and the methods and analysis plan were preregistered (https://aspredicted.org/ymhq-44n8.pdf) on July 8, 2023, prior to data collection, which began on July 10, 2023. The only slight deviation from the preregistered analyses is that we decided to exclude all responses with a response time greater than 10 s. This choice did not affect the conclusions of the study. All other experiments in this article had the same exclusion criterion. At a reviewer's request, we also added unregistered analyses comparing the different trial types. Materials: The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. Data: All primary data are publicly available (https://osf.io/j3vq7). Analysis scripts: All analysis scripts are publicly available (https://osf.io/tuja8). Computational reproducibility: The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 2 disclosures. Preregistration: The research aims and hypotheses, methods, and analysis plan were preregistered (https://aspredicted.org/pt5g-fvt9.pdf) on August 24, 2023, prior to data collection, which began later that same day. There was one deviation from the preregistered analyses: At a reviewer's request, we also added unregistered analyses comparing the different trial types. Materials: The experiment code (https://osf.io/76ngw) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. Data: All primary data are publicly available (https://osf.io/yqsju). Analysis scripts: All analysis scripts are publicly available (https://osf.io/cg5aj). Computational reproducibility: The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 3a disclosures. Preregistration: The research aims and hypotheses, methods, and analysis plan were preregistered (https://aspredicted.org/gcxs-gvbd.pdf) on October 5, 2023, prior to data collection, which began later that same day. There were no deviations from the preregistered analysis plan. However, we did add a binomial test to support the claim that the majority of participants exhibited the relevant effect. Materials: The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. Data: All primary data are publicly available (https://osf.io/uxq63). Analysis scripts: All analysis scripts are publicly available (https://osf.io/8zkua). Computational reproducibility: The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 3b disclosures. Preregistration: The research aims and hypotheses as well as the methods and analysis plan were preregistered (https://aspredicted.org/gcxs-gvbd.pdf) on October 5, 2023, prior to data collection,

which began later that same day. The only deviation from the analysis plan is that we ran a cross-experiment test between Experiments 3a and 3b that was not explicitly preregistered, though we did preregister that we would predict that, if anything, effects in Experiment 3b would be weaker. We also added information about the number of participants who exhibited the relevant effects, with accompanying binomial tests. Materials: The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/gkm2e/files/osfstorage) are publicly available. Data: All primary data are publicly available (https://osf.io/u5n89). Analysis scripts: All analysis scripts are publicly available (https://osf.io/8zkua). Computational reproducibility: The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

Experiment 4 disclosures. Preregistration: The research aims and hypotheses, as well as the methods and analysis plan, were preregistered (https://aspredicted.org/qdwp-dnv5.pdf) on December 9, 2023, prior to data collection, which began later that same day. We preregistered that we would run a version of this task with colored pentominoes but subsequently realized that no pattern of results from that experiment would be diagnostic. For that reason, those data were never collected. Materials: The experiment code (https://osf.io/kza5t) and all other materials (https://osf.io/8wsek/files/osfstorage) are publicly available. Data: All primary data are publicly available (https://osf.io/q25z7). Analysis scripts: All analysis scripts are publicly available (https://osf.io/u23xc). Computational reproducibility: The computational reproducibility of the results has been independently confirmed by the journal's STAR team.

ORCID iD

Sami R. Yousif https://orcid.org/0000-0002-4126-3072

Supplemental Material

Additional supporting information can be found at our Open Science Framework (OSF) page: https://osf.io/3m2ew/.

Notes

1. Here and throughout this article, we use the term "hole" to refer to a connected loop in a network. This sense of the word is different from the more common usage of the word. Whereas a hole in a wall is usually thought of as an intrusion of the space, holes in networks are functional parts of them. The former has been the focus of most prior work (e.g., Chen, 1982), whereas the latter is the focus of ours.

2. We did not preregister any trial-level exclusions. However, when analyzing the data, we decided to exclude any trials with response times greater than 10 s. This decision was made because, realistically, responding should never take more than a second or two. We note that the results presented here are fully unaffected by this decision. The data could be analyzed without this exclusion criterion, or with any other reasonable criterion, and the results would be unchanged. We chose this arbitrary 10-s cutoff because it is consistent with what we have done in similar work and with the other experiments reported here. This note is included here as a matter of transparency.

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