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How We See Area and Why It Matters

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A large and growing literature examines how we see the visual quantities of number, area, and density. The literature rests on an untested assumption: that our perception of area is veridical. Here, we discuss a systematic distortion of perceived area and its implications for quantity perception more broadly.

Imagine a set of 50 marbles, some red and some blue, strewn across a table. How many more of one color would there need to be for you to estimate, based on a quick glance, which was greater in number? Surprisingly, this simple estimation task may reflect more than mere visual ability; it may predict how well you would do on a standardized math test. Indeed, the ability to visually discriminate numerosity is thought to be supported by an approximate number system – a core cognitive system underlying mathematical thought [1,2].

As a result, much work has been devoted to understanding the perception of number itself – ostensibly because number drives the relationship between number estimation and mathematical thinking. However, this poses a challenge: how do we know that people estimate number at all? If we assume, as in the earlier example, that all marbles are of equal size, you do not need numerical information to solve the task. Instead, your visual system could rely on the overall area/volume of the red versus blue marbles to guess which set was greater in number. This highlights a critical challenge for quantity perception research. If we want to understand any one dimension – say, number – we must equate every other dimension. For example, if you want to know whether people can discriminate 26 red marbles from 24 blue marbles, then you would equate the total area of the two sets. If observers can still tell which set has more marbles, they must not be using area to make that judgment. Indeed, many studies take exactly this approach; they make claims about number by carefully isolating it from other spatial properties. Hundreds of studies later, a clear consensus emerges: not only can numerical information be perceived independently from other dimensions, but it may even be prioritized over other spatial properties [3].

Yet this consensus view depends on a subtle, but critical, assumption – that the perception of other quantities, like area, is roughly veridical. Is it? Here, we address this assumption directly, highlighting a recent model that challenges not only contemporary understanding of area perception, but also our understanding of quantity perception more broadly.

Additive Area

If you wanted to calculate the area of a rectangle, you would multiply length times width. But how does your visual system measure the area of a rectangle? We propose that the visual system approximates area not by multiplying the dimensions of space together but by adding them (Figure 1A–C) [4,5]. As such, we refer to this phenomenon as the additive-area heuristic.

This seems unnatural at first, yet this heuristic provides a simple explanation for many different illusions of area [4–7]. Why, for example, does a square rotated 45 degrees appear larger than the exact same square in its canonical orientation [7]? Additive area provides a simple explanation: the sum of the horizontal and vertical extent of the diamond is greater than the same value for the square. In fact, this heuristic results in massive distortions of perceived space; observers fail to discriminate sets of objects that differ in true area by as much as 30% when additive area is equated (Figure 1D,E).

Consider for a moment what this means for number perception. If area perception is not veridical, what should we make of studies that have accounted for true, mathematical area rather than perceived area (i.e., additive area)? Here, we will answer this question highlighting research areas that are most directly influenced by these findings.

Implications

General Magnitude

Consider Figure 1F. From left to right, the numerosity in each image increases while true, mathematical area is held constant. Yet, observers consistently indicate that the rightmost stimulus has more area than the leftmost stimulus. Why? General magnitude theory (GMT) explains such effects by appeal to a common magnitude representation: the mind fails to independently represent any one dimension of quantity, resulting in congruity effects between them (i.e., a stimulus with more number will be perceived as having more area, and vice versa; [8,9]). Additive area offers another explanation: perceived area is greater in the rightmost stimulus because area perception is not veridical and additive area is greater in that stimulus. Among other reasons, we think additive area provides a better explanation than GMT because additive area predicts area judgments even when number is equated across stimuli; additive area affects area judgments when number is equated, but number does not affect area judgments when additive area is equated [4,10]. In other words, there may be no congruity effects at all; the relevant dimensions may have been improperly measured.

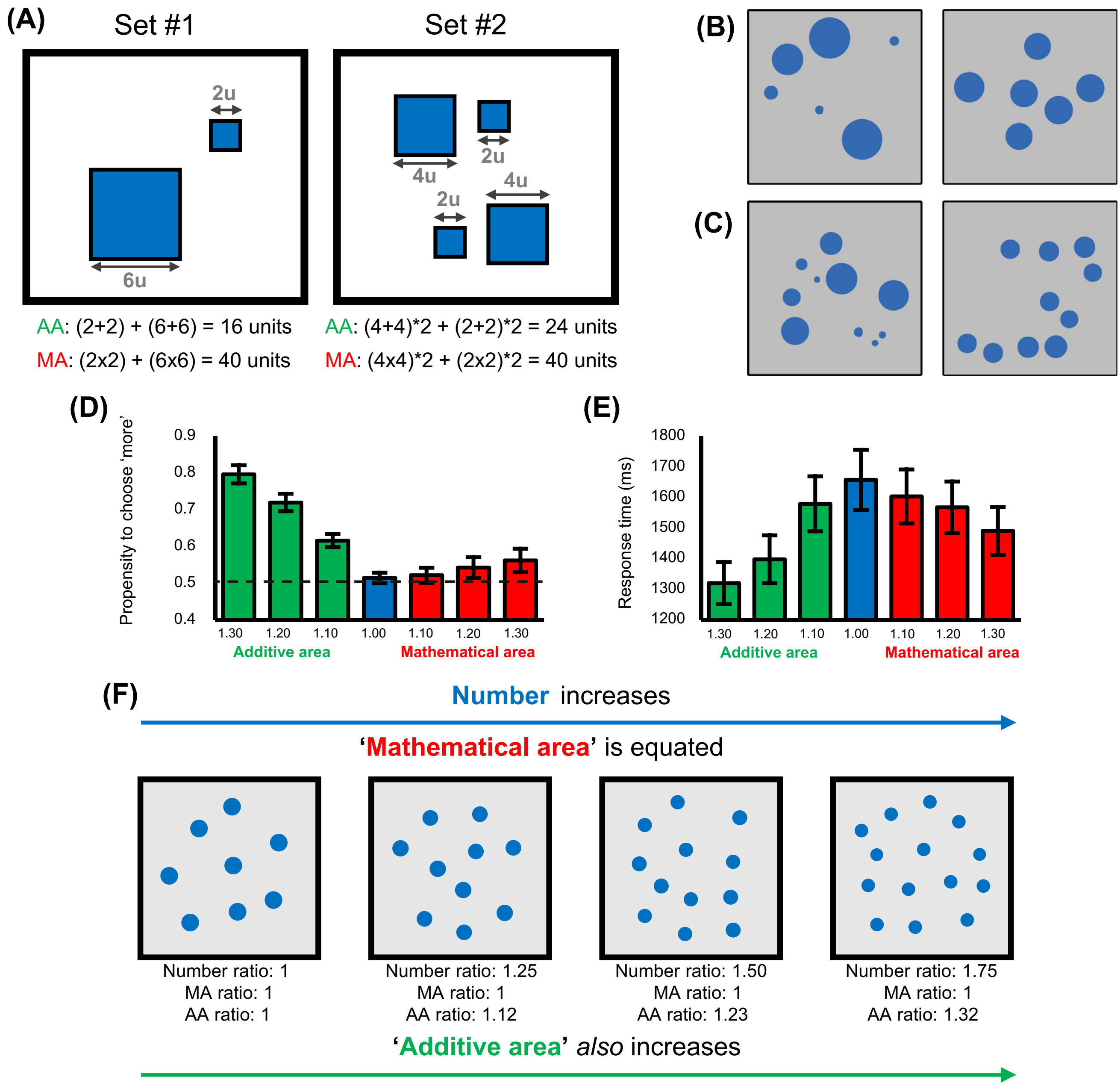

Figure 1. Demonstrations of Additive Area (AA). (A) Visual explanation of the relationship between AA and mathematical area (MA). Note that MA refers to the true, objective area value; yet work demonstrates that AA better captures subjective impressions of area. (B,C) Two demonstrations of sets that are equated in MA but vary in AA. Observers perceive the right sets as having more area than their left counterparts. The right sets have 15% more AA. These stimuli are taken from Yousif *et al.* [7]. (D) Accuracy at area discriminations at different AA:MA ratios; accuracy here represents propensity to choose the stimulus with more, whether it has more AA or MA. Accuracy for the middle ratio was dummy-coded and should be expected to be at chance. These data are from Yousif and Keil [4]. (E) Response times for area discriminations at different AA:MA ratios; data from Yousif and Keil [4]. (F) Visual explanation of the relationship between AA, MA, and number. The ratios displayed below each image correspond to the number:MA:AA ratio compared with the leftmost image. This demonstrates that AA and number are necessarily confounded so long as MA is equated while number varies. As a general rule, the AA ratio will be approximately half the number ratio.

Is Number Special?

As noted earlier, consensus holds that visual numerical information is prioritized over spatial dimensions [8,10–12]. Perhaps the most powerful evidence for this view comes from Ferrigno and colleagues [3]. Participants in that study completed a categorization task in which they could categorize stimuli that varied in numerosity and area. Strikingly, across age, culture, and species, there was a universal bias to categorize based on numerosity rather than area. Yet, as in most studies, only true, mathematical area was controlled – not perceived area. However, additive area (and therefore perceived area) is confounded with number: two displays equated in true, mathematical area but varying in number will necessarily vary in additive area (Figure 1F). Thus, in principle, participants may not have been categorizing based on numerosity at all, but instead on differences of perceived area.

There are other reasons why number may be special. For example, the congruity effects between number and area often favor number, such that number often influences area more than the reverse [9]. Yet, when we manipulate perceived area rather than true, mathematical area, this is not the case: changes in numerosity no longer influence area judgments yet changes in perceived area do influence number judgments [4,10]. In fact, a reanalysis [12] of relevant data [9] revealed exactly this pattern. When additive area was accounted for, area biased children's number judgments more than number biased their area judgments (the opposite of what the authors originally reported). This reanalysis shows that confounds with perceived area are sufficiently large as to be capable of undermining the conclusions of prior work. As such, future work should carefully consider how area is measured/controlled (Box 1).

Box 1. Recommendations for Future Work

To guide future work, we offer two concrete recommendations for addressing area perception. First: collect/store information about additive area and/or perceived area. This practice is exemplified in the work of Aulet and Lourenco [14], which bases area/number comparisons on psychophysical data. Second: be wary of comparing apples to oranges. Simply rotating a square 45 degrees significantly alters its perceived area; diamonds are perceived as having more area than equivalent squares [6]. Although we have proposed that such illusions are explained by additive area, effects like these are still not fully understood. Until they are, comparing area judgments on dissimilar shapes may yield confusing results.

Mechanism

It is tempting to assume that the visual system just counts pixels to determine area. But is that computationally plausible? Consider again that simply rotating a square 45 degrees causes it to appear larger [7]. This challenges the most basic assumption of any pixel counting model – that two things equal in area should be perceived as equal. And this additive heuristic is not merely an anomaly of 2D area perception; it applies equally well to 3D volume perception [13]. This suggests that the mechanism underlying area estimation may in fact reflect a more general perceptual constraint, one that affects all spatial perception. We see this as evidence of a general failure to properly integrate information across spatial dimensions [6]. This raises the question: what other effects of spatial perception may result from this same failure?

Mathematical Competence

Perhaps the most intriguing fact about number perception is its relation to mathematical ability [1,2]. But how does area perception relate to mathematical ability? There is a radical possibility: that the correlations between number perception and math ability may actually be driven by area instead. As we have said, controlling true, mathematical area while varying number necessarily creates a confound with perceived area. As a result, area discrimination, rather than number discrimination, may correlate most strongly with math ability. Although area perception probably does not explain all the variance in the relation between number discrimination and math ability [10], it may be the dominant factor. For the field of quantity perception, this is a key difference: it may affect not only how we think about the mechanisms underlying the perception of number, area, and so on, but how we think about the relation between quantity perception and our core cognitive abilities.

Concluding Remarks

Additive area not only reveals massive distortions of spatial perception; it challenges us to deeply consider the assumptions on which our science rests. So far, work embracing this new view of area perception has highlighted how several findings are at risk if area perception is (properly) considered. Future studies must address these potentially disruptive results.

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Declaration of Interests

None are declared by the authors.

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