Short Report

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A Ubiquitous Illusion of Volume: Are Impressions of 3D Volume Captured by an "Additive Heuristic"?

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Abstract

Several empirical approaches have attempted to explain perception of 2D and 3D size. While these approaches have documented interesting perceptual effects, they fail to offer a compelling, general explanation of everyday size perception. Here, we offer one. Building on prior work documenting an "Additive Area Heuristic" by which observers estimate perceived area by summing objects' dimensions, we show that this same principle—an "additive heuristic"—explains impressions of 3D volume. Observers consistently discriminate sets that vary in "additive volume," even when there is no true difference; they also *fail* to discriminate sets that truly differ (even by amounts as much as 30%) when they are equated in "additive volume." These results suggest a failure to properly integrate multiple spatial dimensions, and frequent reliance on a perceptual heuristic instead.

Keywords

3D perception, perception, perceptual organization, spatial cognition, spatial vision

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In a classic demonstration, Piaget presented children with two identical glasses containing equal amounts of water. Water from one glass was then poured into another taller, skinnier glass, and children were asked which glass held more water. Famously, children select the taller glass—and this is seen as evidence that children fail to understand the conservation of volume between the two containers (Piaget, 1952). But what is the nature of this bias? Why do children perceive the taller glass as having more in the first place?

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Our ability to accurately perceive and interact with the space around us is crucial in our everyday lives—when repositioning furniture, when navigating a busy street, when cooking or purchasing food. Despite the importance of 3D spatial perception, we know surprisingly little about how our visual system perceives 3D size. Although adults can accurately complete Piaget's conservation task when they see liquid transferred from glass to glass, they still make the same mistake as children when they do not have this prior knowledge; that is, adults, like children, indicate that taller/longer objects are greater in area/volume (e.g., Frayman & Dawson, 1981; Holmberg & Holmberg, 1969; Raghubir & Krishna, 1999). While some work has *characterized* this illusion, there is no general explanation (to our knowledge) for why such illusions exist.

Illusions of Area

To understand how we perceive 3D volume, it may be helpful to understand how we perceive 2D area. Classic work proposes "scaling models" of area perception whereby perceived area is equal to true, mathematical area scaled to some exponent less than one (such that larger objects are perceived as smaller than they should be; Ekman & Junge, 1961; Nachmias, 2008, 2011; Stevens & Guirao, 1963; Teghtsoonian, 1965). Other work has emphasized how other stimulus dimensions like number and density contribute to perceived area (Barth, 2008, DeWind et al., 2015; Odic et al., 2013; Tomlinson et al., 2020; Yousif & Keil, 2020), or has tried to explain area perception by appeal to specific illusions (e.g., Carbon, 2016). Often, these different research areas offer divergent explanations for area perception.

Recently, however, it has been suggested that many known illusions of area can be explained by an "Additive Area Heuristic" (Yousif & Keil, 2019). This view posits that perceived area is equal to the *sum* of objects' dimensions rather than the *product*. Imagine that you have a 4×4 square, for example. This square would have a true, mathematical area of 16 (4 times 4), but "additive area" of 8 (4 plus 4). You may also imagine a 6×2 rectangle which has a "true area" of only 12 (6 times 2), but an "additive area" of 8 (6 plus 2). In reality, the 4×4 square has more area. However, according to the "additive area" view, your visual system may perceive these two as being approximately equal (because their "additive areas" both sum to 8). Across many different paradigms, using many different shapes and procedures, variance in "additive area" predicts human area judgments better than variance in true, mathematical area—even to the extent that observers are *unable* to discriminate differences in true, mathematical area as large as 30% when "additive area" is equated (Yousif et al., 2020; Yousif & Keil, 2019).

These illusions of area might arise from a failure to properly perceive or integrate multiple spatial dimensions (Carbon, 2016; see also Yousif et al., 2020). If true, 3D volume perception should reveal the same failure. In other words, we may expect that 3D volume perception also abides by an "additive heuristic."

Current Study

Here, we expand the concept of "additive area" to "additive volume"—asking whether the same perceptual heuristic explains impressions of size for both 2D and 3D objects. To do so, we had observers complete a volume judgment task on 3D printed stimuli. We created sets of 3D printed cubes and spheres that varied in "additive volume" (while "mathematical volume" was equated) or "mathematical volume" (while "additive volume" was equated; see Yousif & Keil, 2019, Yousif et al., 2020 for additional examples of this design).

To understand how "additive volume" and "mathematical volume" can be dissociated, see Figure 1. Which looks like it has more: the three green cubes or the single blue cube? In fact, the two are equal. The single cube has a "mathematical volume" of 1,000 units ($10 \times 10 \times 10$). Each of the smaller green cubes has a "mathematical volume" of 343; in total, they have about the same "mathematical volume" (1,029 units) as the single $10 \times 10 \times 10$ cube (1,000 units). However, the "additive volume" of the three cubes is much larger than the "additive volume" of the single blue cube: Each shape has an "additive volume" of 21 units, for a total of 63 units, compared to an "additive volume" of just 30 units for the single blue cube. In other words, although the three smaller cubes are approximately equal in volume to the larger cube, they vary greatly in "additive volume." This view predicts that people should perceive the three smaller cubes as greater in volume (as you may experience for yourself).

If perceived volume is veridical, we should expect that observers indicate that a set with more "mathematical volume" has more volume—but they should be unable to discriminate sets which vary only in "additive volume." However, if perceived volume is captured *solely* by "additive volume" (as has been shown for 2D area judgments; e.g., Yousif & Keil, 2019; Yousif et al., 2020), observers should indicate that a set with more "additive volume" has more volume, but that sets varying in "mathematical volume" are indistinguishable. Finally, the results may be mixed: Observers may use both "additive volume" and "mathematical volume" to make volume judgments—or they may use "additive volume" under some, but not all circumstances. This would suggest that while observers do rely on an "additive heuristic," there are limits to its use.

Method

This experiment borrows a design from studies on "additive area," in which observers were shown stimuli that consisted of two set of shapes (Yousif et al., 2020; Yousif & Keil, 2019). The two sets of shapes varied in either total additive volume (AV) (while mathematical volume was equated) or total mathematical volume (MV) (while additive volume was

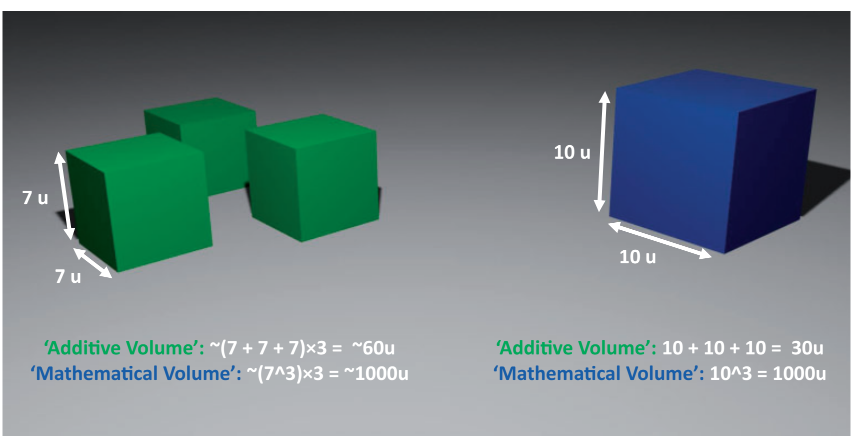

Figure 1. An illustration of the concept of "Additive Volume," and a visual explanation of how it can be dissociated from "Mathematical Volume." Here, the green objects have approximately the same true, "Mathematical Volume" as the blue object. However, most observers perceive the green as having more volume. Although these stimuli are not the exact ones used in our experiment, they are representative to those that we used. Observers saw real, 3D forms of the objects rather than 2D renderings.

held constant). Their task was simple: Observers were asked to judge which of the two sets "looked" like it had more cumulative volume. The goal of this design is to determine whether "additive volume" or "mathematical volume" (or both, or neither) best captures impressions of 3D size.

Participants

A total of 40 participants from the New Haven, Connecticut community completed the experiment in exchange for monetary payment. Each individual consented prior to participation and the experiment was approved by the Institutional Review Board at Yale University.

Materials

The designs of the stimuli used in each experiment were generated via TinkerCAD and produced by a MakerBot 3D Printer. The color of each individual object was selected by availability and was controlled, such that the sets of shapes in each stimulus pair were matched. The shapes were colored either yellow, purple, orange, or green. The stimuli were arranged on a shelf in a well-lit lab space.

In each experiment, there were two stimulus pairs (i.e., four sets of shapes total). One stimulus pair varied in AV while MV was equated (MV ratio: 1.00, AV ratio: 1.29); the other stimulus pair varied in MV while AV was equated (MV ratio: 1.28, AV ratio: 1.00). (Note that these ratios did not perfectly mirror one another; this is because the stimuli were generated pseudo-randomly and there are mathematical limitations on the extent to which these two dimensions can be dissociated.) For one set of observers, the objects were cubes. For another set of observers, the objects were spheres. Both sets of objects for both pairs consisted of six cubes/spheres. The AV and MV volume ratios for the stimuli in each experiment were matched. (Note that for our purposes, the AV of a sphere is equal to 3 times the diameter; that is, a sphere with diameter of 5 units would fit perfectly inside of a cube with side length 5 units; these two objects would have the same additive volume because their spatial extent on each side in the same, e.g., 5 units + 5 units + 5 units.) See Table 1 for the specific stimulus parameters. We checked to ensure that all shapes were printed accurately; the true dimensions of each shape were always within 0.5 mm of the stated value.

Table 1. Stimulus Dimensions.

Pair 1: AV equated/MV varies				Pair 2: AV varies/MV equated			
Set 1		Set 2		Set 1		Set 2	
Object no.	Length	Object no.	Length	Object no.	Length	Object no.	Length
1	31 mm	1	20 mm	1	10 mm	1	30 mm
2	32 mm	2	25 mm	2	12 mm	2	32 mm
3	34 mm	3	35 mm	3	20 mm	3	32 mm
4	38 mm	4	45 mm	4	40 mm	4	36 mm
5	40 mm	5	50 mm	5	50 mm	5	40 mm

Note. AV = additive volume; MV = mathematical volume.

Procedure

The task was administered in-person. Participants stood a few feet from both pairs of stimuli, which were each displayed on a shelf. Participants were instructed to observe the pairs in a counterbalanced manner, such that half the participants were directed to the pair which varied in AV, then the pair that varied in MV, and vice versa. After directing the observers' attention to a stimulus pair, the experimenter asked: "Which group of cubes/spheres *looks* like it has more volume?" The experimenter then asked a follow-up question: "How much more volume does that group have, compared to the other?" Participants were instructed to provide their responses in the form of a percentage. If participants were reluctant to provide an answer, they were encouraged to guess to the best of their ability. If participants stated that they were equal in volume, they were nevertheless encouraged to guess which one had more. If they continued to insist that the two sets were equal in volume, that answer was accepted. These questions were then repeated for the second pair. All participants were allowed to approach the objects and view them from different angles if they wished, though they were asked to refrain from touching them.

Results

The results are shown in Figure 2. First, we analyzed the cubes. For the set which varied in AV, 17 out of the 20 observers reported that the set with more AV appeared to have more volume, despite being equal in MV ($p = .001$, binomial test). On average, observers reported that the cubes with more AV had 14% ($SD = 2.8\%$) more volume—$t(19) = 4.96$, $p < .001$, $d = 1.11$; see Figure 2A. In contrast, only 13 out of the 20 observers reported that the set

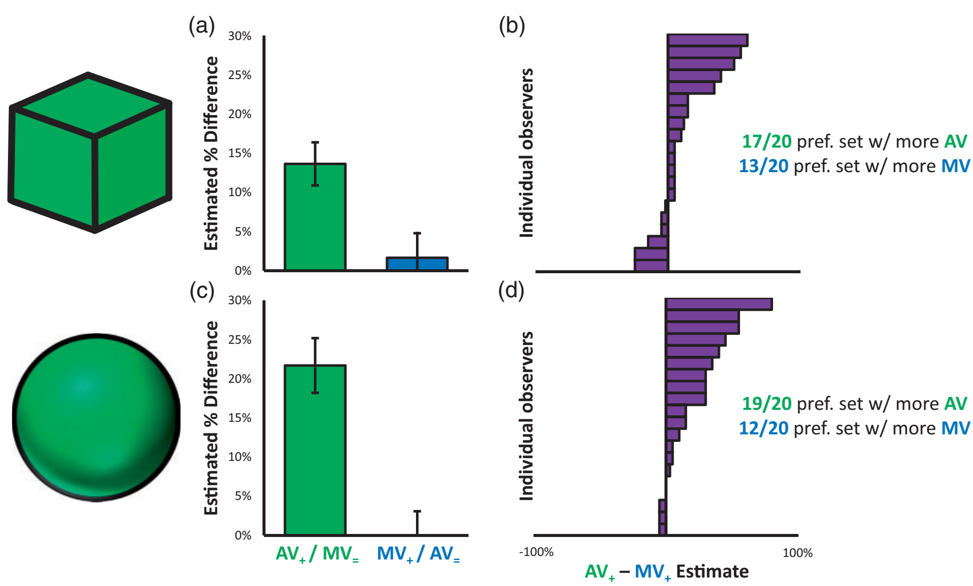

Figure 2. Results. Panels A and C depict the mean estimated differences between the sets in each pair; each bar corresponds to a different stimulus pair. The pair represented by the green bar varied in AV (AV_+/$MV_=$); the pair represented by the blue bar varied in MV (MV_+/$AV_=$). Panels B and D depict the *difference* in mean estimates between the two conditions. Bars to the right of the axis represent subjects who perceived a greater difference in the AV condition (AV_+/$MV_=$) compared to the MV condition (MV_+/$AV_=$). Error bars represent $\pm$ 1 *SE*. AV = additive volume; MV = mathematical volume.

with more MV appeared to have more volume, despite the fact that they varied in MV by 30% ($p = .13$, binomial test). On average, observers reported that the cubes with more MV had only 2% ($SD = 3.1\%$) more volume; this difference was not significantly different from chance—$t(19) = .53$, $p = .61$, $d = .12$; see Figure 2A. Furthermore, the difference between these two conditions was significant: That is, observers consistently thought the pair which varied in AV varied *more* than the set which varied in MV—$t(19) = 2.19$, $p = .04$, $d = .49$; difference scores for each participant are plotted in Figure 2B.

Next, we analyzed the spheres. For the set which varied in AV, 19 out of the 20 observers reported that the set with more AV appeared to have more volume, despite being equal in MV ($p < .001$, binomial test). On average, observers reported that the spheres with more AV had 22% ($SD = 3.5\%$) more volume—$t(19) = 6.21$, $p < .001$, $d = 1.39$; see Figure 2C. In contrast, only 12 out of the 20 observers reported that the set with more MV appeared to have more volume, despite the fact that they varied in MV by 30% ($p = .25$, binomial test). Observers reported that the spheres with more MV had 0% ($SD = 3.3\%$) more volume; this difference was not significantly different from chance—$t(19) = .06$, $p = .95$, $d = .01$; see Figure 2C. The difference between the two conditions was significant—$t(19) = 4.05$, $p < .001$, $d = .91$; difference scores for each participant are plotted in Figure 2D.

Discussion

Impressions of 3D volume are illusory—and these judgments may be explained by an "Additive Heuristic." "Additive volume" captured variance in volume judgments for both cubes and spheres, whereas true, mathematical volume, remarkably, did not. These data yield two conclusions: (1) our impressions of 3D space are systematically distorted, and (2) there may be a simple explanation as to *how* they are distorted—via an "Additive Heuristic."

We propose that *in general* (i.e., for 2D space as well as 3D space; see Yousif & Keil, 2019) people fail to properly perceive or integrate multiple spatial dimensions at once (see also Carbon, 2016). This view is consistent with classic findings showing that one-dimensional spatial judgments are accurate whereas 2D and 3D spatial judgments are "scaled" (e.g., Ekman & Junge, 1961; Nachmias, 2008, 2011; Stevens & Guirao, 1963; Teghtsoonian, 1965). Unlike scaling models, however, we make a specific prediction about the computation underlying area and volume perception. That is, we suggest the visual system relies on a "shortcut": summing the visible dimensions rather than multiplying them together (hence the "Additive Heuristic"). This shortcut is (a) efficient, and (b) generally accurate enough that objects larger in size will appear larger in size.

Limitations and future considerations

The "Additive Heuristic" does not—and could not—perfectly explain volume judgments. Just as with 2D area perception, we expect that many context-specific factors (e.g., the orientations of objects, their relative positions, etc.; see, e.g., Carbon, 2016) may influence 3D volume judgments. Furthermore, this heuristic is not viable at the extremes. Consider a $6 \times 6 \times 6$ cube. That cube has a "mathematical volume" of 216 units, but an AV of 18 units. Now consider three $2 \times 2 \times 2$ cubes. Combined, they have a "mathematical volume" of 24 units, but the *same* AV of 18 units. In other words, the $6 \times 6 \times 6$ cube has nine times as much true, MV as the three $2 \times 2 \times 2$ cubes, though they are equated in AV; does the "Additive Heuristic" view imply that these two should be perceived as equal? In short, no: we think the "Additive Heuristic" is just that—a heuristic. We do not mean to suggest that this is the only means of spatial perception, or that this model will perfectly capture behavior under all circumstances. Thus, a question remains about how generally this heuristic can be applied. Just as we can overcome cognitive heuristics (upon reflection, or otherwise), we believe that observers can use other information to overcome this perceptual heuristic, as well.

The effects observed here are robust: 36 of the 40 participants showed the basic AV effect. However, we see these data as nevertheless preliminary. More work will help us understand the limitations of this heuristic—or whether some other model entirely may better capture volume judgments. Here, the aim is to document a clearly discernible illusion; we remain open-minded about the limitations of the "Additive Heuristic" as an explanation for this illusion.

Conclusion

Our world is spatial, but our visual system does not accurately perceive that space. Instead, we *approximately* perceive that space—seeming to rely on an "Additive Heuristic" to make even the simplest, most foundational spatial judgments (e.g., area and volume judgments). Furthermore, these illusory percepts of volume may be analogous to illusory percepts of area, perhaps suggesting a fundamental inability to perceive or integrate multiple spatial dimensions.

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